THE 51st—23rd INTERNATIONAL—RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS

22 October—2 November 2020

1. Compartmental and agent-based models are the two major mathematical frameworks to describe epicemic models. The former one investigates the number of individuals (or proportion of the population) according to their disease status by a set of ordinary, deterministic differential equations. One of these models is the so-called SIS model. In these equations, S and I refer to the number of susceptible and infectious individuals, respectively. The second S indicates that repeat infections are possible. In contrast to the SIR models wherein individuals become later immune or recovered (R).

   Although the current pandemic situation (COVID-19) can hardly be examined with this model, since we still do not know exactly whether a person can be re-infected, the qualitative dynamical behaviour is encoded.
   The SIS model reads as follows

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{\beta SI}{N} + \gamma I,$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\beta SI}{N} - \gamma I,$$

   where the infectious rate, β, controls the rate of spread which represents the probability of transmitting disease between a susceptible and an infectious individual. Recovery rate, γ, is determined by the inverse of the average duration of infection. The total population $N = S+I$ is constant. Investigate the epidemic evolution with respect to different pairs of (β, γ). If the basic reproduction number $R = \beta/\gamma$ is larger than 1 initially, different kinds of intervention should be applied in order to reduce the number of infectious individuals. Such an option is, for example, a smooth, monotonic decreasing, time-dependent transmission rate, $\beta(t)$ (mimicing voluntary or compulsory changes in population's defensive behavior, distancing, wearing a mask) with time scale comparable to the average duration of infection or even 1–2 magnitude longer. The intervention is effective when $R \approx 0.9$ has been achieved. Show numerical evidences for various scenarios.

   (Tamás Kovács)

2. Climate change (along with other phenomena) has directed the scientific attention to physical systems subjected to temporal parameter change. The damped, driven harmonic oscillator is a paradigmatic example of physics in which the sinusoidal driving with fixed amplitude correspond to annually periodic, stationary climate in the climate analogy, In the spirit of this analogy, investigate the case when the driving amplitude changes (grows or decreases) in time linearly, starting from some initial value. Just like the long-time behavior of the well-known case is characterized by a sustained periodic motion (attractor), there also exists an oscillatory motion to which all initial conditions converge in this case as well, whose amplitude however now becomes *time-dependent*. Determine the parameters of this oscillatory dynamics and display the motion. Is the tendecy for resonance inherited during the 'climate change'?

   (Dániel Jánosi and Tamás Tél)

3. Loránd Eötvös was designing in one of his early experiments a dynamic method to measure the gravitational constant. He has placed two equal, square-based cuboid lead blocks such that their opposite faces are parallel, and their distance is to be equal to the $a = 30$ cm long base edges (see corresponding figure). The height of the cuboids was $h = 2a$.

   In between the lead blocks the torsional balance was fitted, such that the supporting rod is precisely in the center between the bricks. The rod, with negligible thickness and weight, and length $2l$ (evidently, $l < a/2$) held two balls of same mass at its end. The center of the rod was fixed to a torsional wire, so rotational oscillations were possible in the horizontal plane. During the experiment, he has measured the oscillation period at small rotation angles around the two equilibrium positions of the torsion balance.

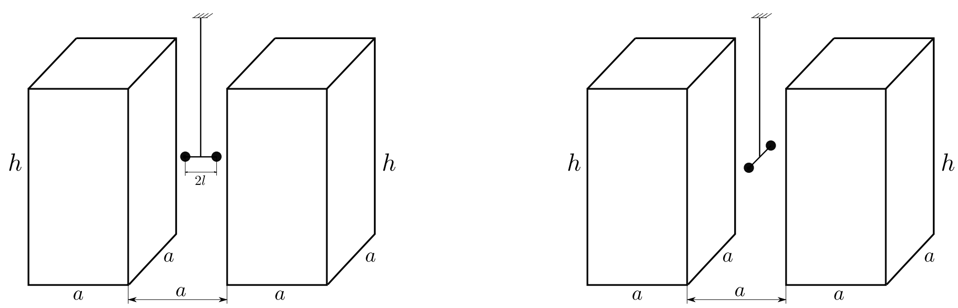
   Let us denote the period of 'transversal' oscillation around the equilibrium state of the rod perpendicular to the sides by T_t (see left panel of figure), and the 'longitudunal' oscillation period when the equilibrium configuration is set to be parallel with the block faces by T_l (see right panel of figure). Note that historically the 'period' time was considered to be half of the present meaning. According to the theoretical calculation, the gravitational constant f could be determined in this measurement of Eötvös using the following expression:

$$\frac{1}{T_l^2} - \frac{1}{T_t^2} = \frac{13.427}{\pi^2} f \varrho (1 - \varepsilon),$$

   where ϱ is the density of the homogeneous lead blocks, and ε is a correction depending on the geometry of the rod and taking into account the finite size of the blocks. Let us show that the numerical value in the formula does not depend on the size of the prisms, but only on the ratio of the length of the edges h/a (which is fixed in this case), whereas it does depend on the arm length of the balance.

   It happens that one can not find any reference to the actual value of l in any of the publications by Eötvös. So the question arises: at which value of l do we get the specific numerical value 13.427? How does this number in the formula depend on l in the interval $0 < l < a/2$?

   Remark: Loránd Eötvös was planning to make the measurement more precise, so that the experiment would be performed "instead of using the unreliable lead bars, with the truly homogeneous mercury" in vacuum. This measurement however have never taken place.



(József Cserti)

4. Consider the following system: Bodies of different mass are in rest initially. A body with mass M is vertically thrown with velocity V_1 by a machine. When the body falls back to its initial height with velocity V_1', the machine gives all of its kinetic energy to another body with mass $M/\sqrt{1.1}$, throwing it up vertically as well. The process repeats, the energy is always given to a body with mass of $\sqrt{1.1}$ times smaller than the mass of the previous body.

   The drag force is proportional with the square of the velocity, and the initial velocity V_1 happens to be the terminal velocity V_∞ associated with the first body and the quadratic drag-force.

   Express the final velocity V_{101}' of the body indexed as $N = 101$.

(Ákos Gombkötő)

5. We have measured the temperature in a room, presented in the attached table (see here), while heating at a steady maximum rate (power), and while not heating at all (after turning the heating off). In the heating case, the outside temperature was a constant 7.4 °C, while in the cooling case it was a constant 2.8 °C. The room has 50 % of its walls (and floors and ceilings) bordering the neighboring apartments, in where the temperature is kept constant. The other 50 % of walls are outside walls. Questions:

   1) Explain qualitatively the shape of the two curves as a function of time!

   2) Fit appropriate functions to the curves! What is the interpretation of the fit parameters?

   3) What is the temperature in the apartment of the neighbours?

   4) During the heating curve, the heating power was constant, and the cost of the heating in this period was 1 EUR. Normally, at the same outside temperature, this room is kept at a constant inside temperature of 20 °C. How much does the heating cost per month?

   5) What is the maximum temperature that can be sustained in this room, if we keep the heating power (the same what we used when measuring the temperature curve) turned on all the time, if the outside temperature is 0 °C?

   6) At 2.8 °C outside temperature we are heating the room every day for x hours, and let it cool for $24 - x$ hours, where $0 < x < 24$. When the heating is on, we use a regular thermostat which turns on and off at 20 °C. What will be our monthly heating cost as a function of x, relative to the cost for the $x = 24$ case? Make a plot!

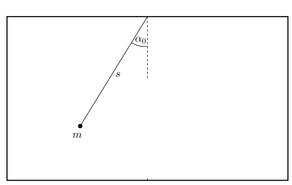
   7) Assume that the outside temperature changes sinusoidally with a period of one day as a function of time. Then the inside temperature will also change sinusoidally as a function of time, but with smaller amplitude and with a delay (phase), without heating. How much is the amplitude reduction factor and how much is the delay?

   (Gábor Veres)

6. Recently many mopping buckets contain an attached top structure to squeeze water out of the cloth:

   The mass distribution of the bucket is then asymmetric: hanging the empty bucket, the angle between the top edge and the horizontal plane is α_0. Let us model the bucket with a cuboid with sides 30 × 25 × 25 cm, mass of m = 750 g, with its center of mass is at a distance of s = 15 cm from the top horizontal edge, in a direction which is at α_0 = 0.16 radian relative to the vertical:

   What is the inclination angle α of the bucket relative to horizontal, if the water level is h, measured along the higher near-vertical edge? At which water level h will the bucket approach a near horizontal attitude, to a precision of 0.01 radian? Wherever useful, let us assume that α is small!

   (Tamás Tél)

7. The solution of the Newton equation, $m\ddot{x} = F(t)$ (where $F(t)$ depends only on time t) can be expressed with an integral operator as,

$$x(t) = \int_{-\infty}^{\infty} G(t - t')\, F(t')\, dt'.$$

What is the kernel $G(t - t')$ of the integral operator?

(Géza Tichy)

8. Two spherical celestial bodies, of masses m_1 and m_2 and moments of inertia Θ_1 and Θ_2, are orbiting around their common center of mass on a circular trajectory. Their distance is R, which is much larger than the size of the objects. The rotation axis of both bodies is perpendicular to the orbital plane, the angular velocities are ω_1 and ω_2 respectively, both larger than the orbital angular velocity Ω.

   Very long time passes, and the two objects become tidally locked, that is, they turn permanently the same side towards each other.
   Which fraction of the original mechanical energy of the system has been dissipated as heat due to the tidal heating during the process, until complete locking? Give a first order approximation. We can assume that the system is isolated from the rest of the world, with no exchange of energy or angular momentum. The bodies are orbiting around each all along on a circular trajectory.

(Gyula Dávid)

9. Let us study in detail the process leading to tidal locking. Assume a planet orbited by a single moon on a circular trajectory of radius R. The rotational axis of the moon is perpendicular to the orbital plane, the initial angular velocity ω is much larger than the orbital angular velocity Ω. The mass of the planet M is much larger than the mass of the moon m. Let us also assume that the shape change of the planet due to the gravitational effects from the moon is irrelevant. In contrast, the moon is distorted to a tri-axial ellipsoid due to tidal forces, keeping its volume constant. Let us consider that the mass distribution of the moon is homogeneous.

   The tidal wave on the moon is not pointing precisely in the direction of the planet causing the tidal forces, but features a small angle delay. Instead of taking complicated details of elastic deformation into account, let us make a simple approximation: the tidal wave follows the gravitational effect with a constant time delay τ. This time constant τ is much shorter than the rotational period of the moon. Correspondingly, let us assume that the amplitude of the tidal deformation wave is proportional to the tidal force acting on the moon (whic scales as $1/R^3$).

   a) Derive the differential equation governing the change of the moon's rotational angular velocity, assuming that the distance from the planet and the orbital period is constant. Solve this equation.

   b) Now let us take into account the effect that due to the torque excerted on the tidal wave, the orbital period and the orbit radius of the moon change. Write down the coupled differential equations for the rotational ω and orbital Ω angular velocities! Eliminating Ω and introducing well suited parameters, derive the resulting equation for ω only (which does not need to be solved). Let us study under which conditions can we get back the equation in part a).

   c) Describe the more general problem. Let the two masses be comparable. Let both the planet and the moon be deformable, but the formation time constants (delays) should be set different τ_1 and τ_2. The amplitudes of the tidal waves is proportional to the attractive tidal force between the bodies, but the coefficients of this proportionality are different. Assume that the rotational axes of the moon and planet are parallel, and both perpendicular to the orbital plane. The initial rotational angular velocities are ω_1 and ω_2 respectively, both larger than Ω. Write down the coupled equations for the change rate of $\omega_1(t)$, $\omega_2(t)$ and $\Omega(t)$ (which, again, does not need to be solved).

(Attila Gohér and Gyula Dávid)

10. **Sorry, the Problem 10 has been withdrawn (26th Oct 19:52 CET)**

11. The thunder after the lightning starts with a strong clap and the clap is later followed by a long, deep murmur. Explain the effect using the properties of the sound propagation!

(Géza Tichy)

12. Consider the core of the Earth as a homogeneous spherical rigid body of radius R and density ρ_m, which is surrounded by fluid of density ρ. We can assume that the equilibrium point for the core is when its center of mass is at the geometrical center of Earth. Derive the equation of motion of the core for small oscillations.

(Ákos Gombkötő)

13. Two identical point-like magnetic bars are placed on a table with magnetic dipoles aligned on the plane of the table and a) in the opposite direction to each other and perpendicular to their separation direction, and b) in the same direction and parallel to their separation direction. One of the bars is fixed and the other one can move without friction on the table.

    The orientation of the magnetic dipoles does not change during the motion. Calculate the time the moving magnet does take to hit the fixed magnet owing to the attractive force between the two magnetic bars. Assume that the interaction between the two bars can be modeled by magnetic dipole-dipole interaction.

(József Cserti)

14. Two homogeneous, spherical conducting particle, of identical radius and material (unobtainium) collide head-on. The relative velocity is great enough, that the Q charge, distributed evenly on the surface of body 1. does not induce noticable charge-distribution on the electrically neutral body 2. Due to a special property of unobtainium, there is no discharge between the two bodies, currents are only present during mechanical contact. In the process of collision, the elastic deformation takes place on a negligible timescale, during most of the contact both body deforms into a hemisphere and form a single spherical conducting body together. The process of charge-relaxation progresses for T time. After $t = T$ the mechanical contact ceases, and the charge distribution along the plane that separates the two hemispheres will be shared equally between the two bodies.

    At $t = 0$, half of charge Q will be distributed evenly at the outer surface of body 1. After collision, the charges on the bodies are distributed with ratio 3:2 between the spheres. Our only question is the duration of the collision.

(Ákos Gombkötő)

15. On a January morning, a university student prepares for his thermodynamics exam on the following day. He has been wondering recently that entropy has the same unit as heat capacity, but during preparing his tea, he does not feel like thinking about it too deep. However, as he mixes a cold, 16 °C tea with the same amount of warm, 88 °C tea, the following question emerges: how much fraction of the total heat capacity of the tea appears as entropy increase during mixing? Let us help him! Assume that the specific heat of tea does not depend on temperature.

(Gyula Radnai)

16. The slits of an optical grating are not exactly equally wide due to manufacturing reasons. While the distance between the center of two neighboring slits is constant (the grating constant d), the width of the slits fluctuates randomly. The width of the slits is a Gaussian random variable of expectation value a and standard deviation σ_a.

a) Find the diffraction pattern of the grating if a laser beam of wavelength λ falls at normal incidence on it, and the number of slits illuminated is N.

b) How would the diffraction pattern change, if the width of the slits a was constant, but the distance between the neighboring slits was a Gaussian random variable of expectation value d and standard deviation σ_d?

(Máté Vigh)

17. A monochromatic parallel light beam is directed to a high resolution transmission optical diffraction grating with vertical grooves. Then, we rotate the grid around the center vertical grating as axis with the same angle as the n-th deflection order in the original setting. Which are the diffraction directions from the grating in this case?

a) In what directions do the light beams emerge in this case?

c) Define the relation between wavelength, grating period, and rotation angle, when only the 0-th (non-diffracted) order and one of the first order (left or right) is transmitted!

d) Determine the ratio of the maximum and minimum wavelength, for which the above condition can be fulfilled for a given rotation angle.

(Gyula Radnai and Dezső Varga)

18. The equation of motion of a relativistic point particle moving in an external field is $d(Mu_k)/d\tau = F_k$, where τ is the proper time, u_k is the vector of four-velocity, M is the rest mass of the particle, and F_k is the four-force of the external field. Study the motion of the particle in the static central symmetric field of a fixed center. Calculate the orbital 3-velocity of the particle, and the revolution time (expressed by reference time t and by proper time τ) as functions of the radius r of the particle orbit. Investigate

a) the case of the Higgs field where the attractive force of the center is $F_k = g\,\partial_k\Phi(r)$ with coupling constant g, and

b) the case of Nordström gravity theory, where the 'coupling constant' is the rest mass M (as it is ordinary in gravity theories): $F_k = M\,\partial_k\Phi(r)$. In both cases the field $\Phi(r)$ is a four-scalar field with power-like dependence on radius r: $\Phi(r) = -K/r^N$ with positive constants K and N.

c) Recent theories of particle physics assume that the rest mass of some elementary particles is due only to the Higgs field Φ. What is the orbital speed of such particles as a function of the radius R?

(Gyula Dávid)

19. A one-dimensional quantum particle of mass m, in a linear potential Fx, is enclosed into a box of length L, with hard walls (equivalent to a quantum ball vertically bouncing between the the floor and the ceiling). Starting from the stationary Schrödinger equation, taking into account the boundary conditions, write out the equation determining the energy eigenvalues and solve it numerically. Plot the lower levels as functions of the size of the box, and illustrate the stationary wave functions. Given the asymptotes of the of the functions in the expression for the energy eigenvalues, give the simpler equation for the higher levels. Perform also semiclassical quantization, then compare it to the aforementioned approximate formula and, numerically, to some levels of from the exact equation.

    Further questions: a) Give the levels explicitly for small L.

    b) By what parameters will the ground state energy equal FL? (This is the case when the classical ball just reaches the ceiling.)

    c) Show that, in the case L is smaller than the threshold value in (b) then all levels are higher than FL.

    d) Write out the semiclassical wave functions explicitly and compare them graphically to some exact ones – when do we get a good agreement?

    e) Write out the wave functions for small L. These, too, should be visually compared to the exact ones.

(József Cserti and Géza Györgyi)

20. A harmonic oscillator is in the Kth energy eigenstate. Then the system is abruptly changed: the frequency of the oscillator is multiplied by factor $e^{2\gamma}$, where γ is an arbitrary real number. Express the state vector at the moment of the break and by time t after the break using only the emission operator and ground state vector of the new system. Give the answer as well if the initial state of the system is not an energy eigenstate but a coherent state characterized by an arbitrary complex number β.

    To know the probability to find the system in the nth energy eigenstate of the new Hamiltonian you have to calculate infinitely much matrix elements. Construct a generating function with two variables which gives you these matrix elements by simple derivation. Calculate this probability in the special case $n = K$.

    Study the limiting case when the parameter γ tends to zero.

(Gyula Dávid)

21. The electron specific heat of a metal is calculated in every textbook at constant number of electrons, i.e., when the sample is isolated. In this calculation the Sommerfeld expansion is applied. Using the same method, find the electron specific heat at constant Fermi energy, i.e., when the sample is grounded.

(Géza Tichy)

22. In the process of adiabatic demagnetization, paramagnetic salt is used. After the application of magnetic field, vacuum is applied to sure the adiabatic conditions and the field is switched off. The salt cools dawn from kelvin to about millikelvin. Let the paramagnetic salt modeled by independent magnetic dipoles of spin-$\frac{1}{2}$. What is the temperature vs. magnetic field function during the adiabatic cooling? What can we say in case of paramagnetic salt with higher spin?

(Géza Tichy)

23. Tight-binding models on lattices consist of hopping terms on a lattice. Suppose that a lattice extends on the xy-plane, and a homogeneous magnetic field of strength B_0 is pointing in the z-direction. In this case a general hopping operator can be written as

$$\hat{T}_{m,n} = -\sum_{\mu,\nu} J_{m,n} \exp(i\theta_{\mu,\nu}^{(m,n)}) \hat{c}^{\dagger}_{\mu+m,\nu+n} \hat{c}_{\mu,\nu},$$

where $J_{m,n}$ denote hopping parameters, $\hat{c}^{\dagger}_{\mu,\nu}(\hat{c}_{\mu,\nu})$ denote the creation(annihilation) operators for a particle at lattice site indexed by μ, ν.

A typical model is situated on some type of lattice (square, triangular, etc.), and the Hamiltonian is a sum of terms $\hat{T}_{m,n}$ for different m, n and their Hermitian conjugates (for example, $m = 0$, $n = 1$ is hopping between nearest neighbors in the y-direction, etc.). The phase $\theta_{\mu,\nu}^{m,n}$ denotes the line integral of the vector potential starting from the lattice site indexed by μ, ν to the one by $\mu + m, \nu + n$,

$$\theta_{\mu,\nu}^{(m,n)} = \int_{\vec{R}_{\mu,\nu}}^{\vec{R}_{\mu,\nu}+\vec{R}_{m,n}} \vec{A} \cdot d\vec{r},$$

along a straight line.

We define the lattice derivative of function $f(\mu, \nu)$:

$$\Delta_{m,n} f(\mu, \nu) = f(\mu + m, \nu + n) - f(\mu, \nu).$$

- Under what condition do two hopping operators, $\hat{T}_{m,n}$ and $\hat{T}_{p,q}$ commute?

Let us also define magnetic translation operators of the type

$$\hat{\mathbb{T}}_{m,n} = \sum_{\mu,\nu} \exp(i\chi_{\mu,\nu}^{(m,n)}) \hat{c}^{\dagger}_{\mu+m,\nu+n} \hat{c}_{\mu,\nu}.$$

In the following, consider the square lattice, with nearest neighbor hoppings of strength J in both x and y directions (in other words, only J_{10} and J_{01} are finite).

- Derive the conditions under which the magnetic translation operators commute with the full Hamiltonian of the simple square lattice. (In other words, derive the discrete differential equations between $\chi_{\mu,\nu}^{(m,n)}$ and $\theta_{\mu,\nu}^{(m,n)}$.)
- Find the solutions of these equations.
- Check if the magnetic translation operators form a group, and if not, modify them so that they do.
- Assuming a finite system with periodic boundary conditions (L_x, L_y), construct the finite magnetic translation group.

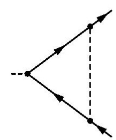
(Balázs Hetényi)

24. The expansion of the Universe is described by the scale function $a(t)$. A free particle passes the origin of our co-moving reference system by velocity v_0 at the moment t_0. What is the velocity of the particle in a later moment t?

(Gyula Dávid)

25. Renormalizability of quantum field theories is inherently connected with the symmetry of the system in question. Renormalizability based on naive power counting is usually not sufficient, the counterterm functional needs to inherit the symmetry of the system, otherwise there might appear divergences that cannot be cancelled via counterterms of the Lagrangian. In case of symmetries that are linear in terms of the field variables, one can easily show that the symmetry of the Lagrangian is inherited by the quantum effective action, thus the counterterm structure is consistent with that of the Lagrangian.

Let us consider a $\phi = (\sigma, \pi^a)$ $(a = 1, 2, 3)$ four component field variable, which is coupled to a massless fermion doublet through Yukawa interaction. The interaction is described by $\mathcal{L}_{\text{int}} = g\bar{\psi}(\sigma + i\pi^a\gamma_5\tau^a)\psi$ in the Lagrangian (kinetic terms are the usual ones), where $\bar{\psi}$ is the Dirac adjoint of ψ, τ^a refers to the Pauli matrices, and γ_5 is the fifth Dirac matrix. It can be shown that the system exhibits $O(4)$ symmetry, which are linearly realized in terms of the fields, thus the structure of the counterterm functional has to be the same as that of the Yukawa term. Let us calculate the δg counterterm at the lowest order of perturbation theory, i.e. calculate the following Feynman diagram (solid oriented lines refer to fermions, while dashed ones correspond to ϕ).



If the dashed external line is σ, then starting from the bottom fermion propagator the contribution of the diagram schematically reads

$$i\delta g = \int_p S(p) \times ig \times S(p)\Big[(ig)^2 + (ig \times i\gamma_5\tau^b)^2\Big]G(p),$$

while if the former is π^a, then

$$i\delta g \times i\gamma_5\tau^a = \int_p S(p) \times ig \times i\gamma_5\tau^a \times S(p)\Big[(ig)^2 + (ig \times i\gamma_5\tau^b)^2\Big]G(p),$$

where S is the fermion, and G is the ϕ propagator. Since $\{\gamma_\mu, \gamma_5\} = 0$ (and thus $\{S, \gamma_5\} = 0$), there is a sign difference in δg between the two calculations. That is to say, in the counterterm functional the operator $\sim \bar{\psi}(\sigma - i\pi^a\gamma_5\tau^a)\psi$ appears, which violates the former $O(4)$ symmetry and is not compatible with the Lagrangian. Where do we make errors when considering the integrals regarding the diagrams? In case of spontaneous symmetry breaking, what other diagram offers the possibility to calculate δg?

(Gergely Fejős and Zsolt Szép)

26. Let's assume that the depth of Lake Balaton was mapped at every point of its water surface and it's described by the function $D(x, y)$, where (x, y) is a point within the contour of the lake shore. If $D(x, y)$ is known accurately, can the following information be derived from $D(x, y)$?

a) What is the most frequently occurring depth, i.e., the mode of depth distribution? b) How can the average depth be defined and formulated? c) How can the density function of depth distribution $f(z)$ be derived from $D(x, y)$ (here $f(z)dz$ is the probability that the value of the depth z in a random measurement falls in the interval $[z, z + dz]$)? Can it be derived at all? d) What is the median depth separating the upper half of the lake volume from the bottom one? e) Where is the lake's center of gravity, if the water density depends on the depth z as $\varrho(z)$? f) What is the median depth separating the upper half of the lake weight from the bottom one? g) How can the 'expected value' of depth $< z >= \int z f(z)dz$ be formulated?

We naturally assume that the lake has a convex underwater ground surface. What if the lake is concave?
If the above questions cannot be handled analytically, what would be the answers for these two special cases: α) The lake has a half sphere shape with the flat surface at the top (bowl)? β) The lake has a half sphere shape with the flat surface at the bottom (bell)?

(Oszvald Glöckler)

27. Inhabitants of the fictious FlatEarth need no belief: they know that they live on a plane surface. They can actually see the edges. The surface is covered with ice, and in the clear air the Perimeter of the huge flatland is apparent to the naked eye. The Perimeter is a uniform square, and the inhabitants live around the center of the Square, to which a mysterious force pulls them back all the time. Philosophers are arguing since ancient times about the precise shape of the planet. No question of course, that FlatEarth (according to the ancient legend) was created by the Flying Spagetti Monster as icecube for his soft drink, but somehow slipped out of his hand and now wanders in space; the question is the precise shape of the icecube. It is agreed that the shape is a square based prism; but how thick? Cubists state that it is actually a regular cube (what else an icecube could be?). According to sheetists, the thickness of the prism is negligibly small relative to the side of the square, as a single plane is sufficient to be popoulated by FlatEarthians. A number of expeditions have been launched to answer the question: the idea was to get to the Perimeter, and after climbing over, they simply measure the side perpendicular to their life plain. Unfortunately, the slippery ice shattered all plans, as the friction is practically negligible. The researchers slided back all the time, as approaching the Perimeter felt like a continuously steepening slope.

    The unsuccessful expeditions were returning to CenterCity, where they met the Braves. These citizens have no fear of the ice fields around the city; they move away from the middlepoint as far as possible, and then pushing themselves perpendicularly, they orbit around CenterCity on skate for a long time. An old sportsmans's experience is that the orbital period is independent from how far one is distancing from the city.

    When scientists, after many disappointments, realized that they will never be able to decide on this ancient question on the plane shape by expeditions to the Perimeter, looked for different methods. Applications were invited to determine the shape using local measurements, to be performed around CenterCity.

    The best project was completed by a physicist group using the acronym 'Idunno'. They bought pendulums, watches, glass cylinders, spades, pickers and levers from the funding. They hung a pendulum on a high pole erected in the center of the city, and other group members measured the orbiting time of the Braves skating around the city. Actually they also started to hack the ice, and found out to their surprise that the whole planet is – except for a thin ice layer – is made of this strange material that they call 'rock'. They could actually measure the density of 'rock'.

    Soon after, the research group announced that they received messages from the inhabitants of a distant celestial body, the latter they call 'SphericalEarth', and they shared the measurement results with this foreign civilization. Soon it turned out that lot of similarities exist between the two planets. The period time of a pendulum in CentralCity is the same as on the north pole of SphericalEarth for the same pendulum length. The Braves, orbiting around CentralCity, take the same time for one turn as the 'equatorial satellite' orbits around SphericalEarth. Finally, it turned out that the 'rock' on FlatEarth had the same density as the average density of SphericalEarth.

    Based on these informations, the researchers could easily determine the precise dimensions of FlatEarth, including the thickness of the prism, thus concluding the ancient philosophical argument. Unfortunately there was considerable political turmoil after the publication of the results, so the findings were not broadcasted to those living on SphericalEarth (us).

    This means that the task to determine the sizes of FlatEarth is left to You, participants of the 2020 Ortvay Competion. The call is to determine the length of the side of the Square, and the thickness of the planet, expressed as multiple of the radius of SphericalEarth. Give exact value. You may afterwards calculate the result numercially as well. Be more fortunate than the unfortunate Perimeter explorers, and more brave than the Braves!

    (József Cserti and Gyula Dávid)

28. In the beginning of the 1970s, when the predecessor of the Ortvay Competition was established by Géza Tichy and János Major, referred to as Physics Students Problem Solving Competition, there was no internet, and not even simple possibility of using copy machine. To this end, the problems were posted on the announcement dashboard, and those standing in front could take notes in handwriting. (Later technological improvements enabled mimeography of typewriter text and handwritten formulas).

    In the autumn of 1970, just few days before the starting of the very first (then not yet called Ortvay) competition there were 36 physicst students (back then, the classes were just that large) crowding in front of the freshly announced problem in the subject of 'Vector calculus', starting in that year. The problem to be noted and solved (with Gaussian elimination) was an inhomogeneous system of six linear equations with six unknown variables. The system of equations was special, with all coefficients and all constant terms being integers of one digit, with either positive or negative sign, but zero was not among these. The teacher of the subject wished to simplify the problem to allow more students to compete, therefore the original problem was inconsistent: it could have been easily proven by Gaussian elimination that no solution exists.
    However, overcrowded corridors do not favour precise information transfer... while attempting to copy the system of equations, every single student made a mistake, such that each one of them wrongly noted exactly one sign of a coefficient (miraculously, all of a different one). Fortunately the constant terms on the right side of the equations were all copied correctly. As a result, an intentionally simple problem instead led to long and complicated calculations, causing a lot of upset feelings (it would have been more interesting to deal with the competition problems).

    The teacher was shocked to see the multiple pages of nasty calculations. He only got a bit relieved when one of the solutions led to the consequence that the equation system is inconsistent (in spite of the fact, as we know, that also this student had one sign incorrect).

    Our question is this: what is the probability, that there is one student among the 36, which gets to this conclusion, despite the sign error?

    Certainly one can assume, that physics students (though not strong at copying) never make errors during the actual calculation, neither principal, nor numerical. ('cause physicist's the BEST, the BEST, the BEST!' — as The Physicists' March sounds).

(Gyula Dávid)

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